The Report Committee for Natasha Marianna Dahanayake
Certifies that this is the approved version of the following report:

The Gazebo Project: A Look Into The Benefits Of Student Discourse In Learning Mathematics Through A Process Of Creating, Critiquing, And Revising A Plan

APPROVED BY
SUPERVISING COMMITTEE:

Supervisor: ______________________________
Catherine Riegle- Crumb

____
Jill Marshall

The Gazebo Project: A Look Into The Benefits Of Student Discourse In Learning Mathematics Through A Process Of Creating, Critiquing, And Revising A Plan

by

Natasha Marianna Dahanayake, B.S. Psy.

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Dedication

This report is dedicated to my Mother who has encouraged and supported me throughout my educational career.

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I would like to acknowledge all of the faculty, staff and fellow colleagues in the UTeach *Engineering* program. The faculty and staff have inspired an excitement for learning and teaching engineering education and I could not have asked for a more enjoyable cohort. I am grateful for this experience and I look forward to the experiences we will share in the future.

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Abstract

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Natasha Marianna Dahanayake, M.A.
The University of Texas at Austin, 2014

SUPERVISOR: Catherine Riegle-Crumb

The Gazebo Project is an open ended, generative, model eliciting project that was designed to allow students to develop their own understanding of fractions rather than receiving direct instruction. The students were placed in three different sections to work on the project, a group section that allowed for collaborative work, a peer tutoring section and an individual section. All students were given a pre-project clinical interview to assess their knowledge prior to beginning The Gazebo Project. They were then separated into one of the three sections for the project. The Gazebo Project charged the students with the task of designing a gazebo that would maximize the amount of seating and minimize the size of the entrance, which needed to be a whole side length. By challenging the students to minimize the entrance they were guided to explore the relationship between side length and number of sides. Upon completion of the project all students were then given a post-project clinical interview to determine the growth in their understanding of fractions. The study suggested that The Gazebo Project was effective in helping students develop their understanding of fractions, but only when the students worked in the group section or the peer tutoring section. The element of student discourse created an environment where students could create, and critique each other’s plan and in the process student discourse contributed to revised thinking. This study challenges educators to consider the benefits of open ended generative activities and discourse in student learning and also encourages the use of regular clinical interviews to assess student reasoning.

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Chapter 1: BACKGROUND

The last two decades have given rise to growing class sizes along with an increased emphasis on multiple choice testing which only measure specific solutions to a problem. These systems have led to a rigid and perhaps oversimplified method of teaching mathematics in schools. Proponents of this rule based learning such as Robert Gagne (1983) focus on the “advantages of making computational operations automatic” (p.7). Rule based learning also allows for classrooms to run like a clock rather than like a living organism allowing for a time efficient model. Students can transfer easily from one school to another with confidence that they will be learning the same material. Though rule based learning models allow for an efficient system there has since been many findings pertaining to the benefits of using open ended, generative, and group activities. For example Walter Stroup suggests that generative activities “provide just such settings where students work to create mathematical and scientific text through shared discourse" (Stroup, Ares, Hurford, & Lesh, 2007, p. 383).

There is now a push away from routine, rule based practices that have led to the creation of numerous charter schools. This shift has led to many other models including model eliciting activities and open-ended generative designs. According to Lesh et al. (2000), model eliciting activities “often go beyond providing documentation about completed learning or problem solving experiences; they also support the productivity of ongoing learning or problem solving experiences. Therefore, thought revealing activities of this type not only help to document development; they also promote development” (p 594). The heart of the model eliciting problem “is for students themselves to develop an explicit mathematical interpretation of situations. That is, students must mathematize situations” (Lesh et al, 2000, p 594).

Through this study I am interested in observing the manner in which students formulate their mathematical concepts in group activities. I hypothesize that when students are given the opportunity to discuss their theories with other students they are challenged by their peer’s critiques and feedback, and these criticisms lead to deepening their understanding of the material at hand.

The purpose of this study is to build upon and confirm observations of the ways that students generate knowledge in mathematics, specifically in the area of verbalizing and formulating a plan in a group dynamic. Through this study I will observe students learning about proportions and scale factors, a 6th grade learning objective, in a group setting. The students will explore proportions and scale factors using a model eliciting, open-ended generative project. I have selected a model eliciting, open-ended generative project because it lends itself well to group work, one of the avenues of learning I will explore in this paper, and because there are many avenues for the students to learn about the topic. The students will not be repeating a process I show them but they will be given the space to manipulate objects and verbalize their observations. The students are also broken down into three main groups. One group is a peer tutoring group comprised of a tutor and a tutee. The other group is comprised of 4 members who all make an equal contribution. The final group will consist of individual students working on the project independently. The purpose of the peer tutoring group versus the group of 4 is to search for any possible differences in the two models. There has been much research on the ways that discourse through group work aids in learning and in creating a plan (Vygotsky, 1978, p. 19, 24). I am interested to see if there is any difference in the ways that discourse plays out when comparing a peer tutoring model, which is more one way by nature, to a group project, which is collaborative in nature.

Chapter 2: LITERATURE REVIEW

<u>History</u>

With the onset of the First World War there came a need for training a "large number of people in the skilled trades, training that must take place in a relatively short period of time". This also meant that "the older and slower apprentice systems were no longer adequate" (Tyler, 1966, p. 54). This efficient educational mentality may have given rise to theories such as Gagne's three phases of math tasks (Gagne, 1983, p7).

Phase 1 is to translate a problem to math. Some problems are presented as word problems etc. Gagne said that the student's first task is to translate the words into mathematics.

Phase 2 is to calculate. Phase 2 is about rules, not about using models. Gagne believed that if the teacher taught the rules correctly the students should be able to apply the rule to other similar problems.

Phase 3 is validation. Validation includes estimation and logic. The students should finally ask themselves if their answer makes sense.

There are remnants of this need for mass training in a short, efficient time in our current educational system. We have many students packed into a room and we want our classes to move like clockwork. We need to convey a certain amount of information in 50 minutes regardless of the questions that come up as a result of the lesson. Gagne's thoughts on direct teaching still dominate many classrooms creating an industrial education system rather than an apprentice model.

<u>Student Discourse</u>

The rise of Gagne's industrial model may have lost some of the magic of student inquiry and open-ended challenges as well as the benefits of group learning. Currently when you walk into a typical classroom in the United States there are many students in a class but they are seated in rows. The children are together yet separate. This group dynamic is rarely utilized. As observed by Walter Stroup et al. (2007), “relative to design, perhaps the most obvious and yet underutilized feature of most classrooms is that they involve groups of people” (p367). Lauren Resnick (1987) also noted that successful schools typically “involve socially shared intellectual work, and they are organized around joint accomplishment of tasks” (p.18). When students work in a group, language becomes a tool for the students to use in problem solving (Vygotsky, 1978, p.19). This leads me to my hypothesis that children spawn understanding through a process of verbalizing their thoughts and receiving critique on those thoughts from peers. Stoup, Ares, Hurford, & Lesh (2007) asserted that “students generate knowledge by repeatedly expressing, testing, and revising their own ways of thinking rather than being guided along narrow trajectories toward cleaned-up and oversimplified versions of their teachers’ understandings” (p. 367). “Students should be free to test and try their hypotheses” (Stroup, Ares, Hurford, & Lesh, 2007, p. 368) and in doing so with their peers they begin to use language as a tool promoting the development of a plan (Vygotsky, 1978, p.24).

Stroup, Ares, Hurford, & Lesh, (2007) suggests that rather than teaching rules and having students apply them we should allow students to test and revise their thinking. This claim that children benefit from formulating even incorrect thoughts in front of their peers builds upon the findings of Lave et al. (1988) who affirmed that “for learners to be successful in the everyday practice of problem solving they must engage in choice, judgment, control processes, problem formulation, and in making wrong choices and wild goose chases” (p 77). Lave et al. also claimed that “children actively construct theories at all times” (1988, p.71). If we recognize that students actively construct theories at all time, we cannot deny the importance of giving them the opportunity to test and correct these theories.

In addition to constructing theories at all times, Vygotsky (1978) also observed that “only when students are interacting does development occur” (p.90). Kieren (2000) comments specifically on the effects of discourse on mathematics. Kieren claims that “mathematics is not simply or even primarily a cognitive phenomenon. It cannot be thought of in isolation from affective aspects… from larger aspects of the cultural phenomenon” (Kieren, 2000, p.229). If “in group problem solving sessions, it is natural for students to externalize ways of thinking that might remain internal otherwise” and if “the goal of learning… may involve developing shared knowledge, rather than only personal knowledge” (Lesh et al, 2000, p.61), then the development of knowledge in the classroom may be cultivated by enabling students to share and foster their ideas in groups.

In studying Stroup and Lave’s work I am challenged to think about ways that we can create an environment for students to test and revise their thinking, an environment where students are able to have incorrect ideas and then transform those ideas as they see the flaws in reasoning. Student discourse about mathematics appears to be an ideal avenue for that sharpening of ideas. As one student suggests an idea, other students are present to shape and challenge the logic and reasoning.

Alternative Learning Models

These findings about the ways children generate knowledge also cause me to consider the manner in which we introduce mathematical concepts to students. If students learn best through having the opportunity to make and correct mistakes, direct teach may be withholding that opportunity. Conversely, open ended generative activities in the way that Stroup et al. (2007) describes, give an opportunity for students to give and receive feedback which causes the examination of assumptions. This is made possible because “the non-prescriptive nature of the task opens up a space in which problem solvers decide the math they find useful in establishing a path from givens to goals” (Carmona & Greenstein, 2010, p. 253), as opposed to nominally generative activities that have a predetermined means and end to a solution (Stroup et al., 2007). Based on these findings, this study will explore how an open ended, generative activity gives students an opportunity to make and correct mistakes as well as the opportunity to make decisions about what math is useful to them, as opposed to telling them how to solve the problem and then asking them to practice. I will examine the impact this freedom to make and correct mistakes can potentially give students in learning mathematical concepts.

In order to observe the impact of allowing students to make and correct mistakes in a non-prescriptive nature, I have chosen to use an open ended, generative activity. Lesh et al (2000) press the value of using Model Eliciting Activities in mathematics. They assert that “the descriptions, explanations, and constructions that students generate while working on them directly reveal how they are interpreting the mathematical situations that they encounter by disclosing how these situations are being mathematized (e.g., quantified, organized, coordinatized, dimensionalized) or interpreted” (Lesh et al., 2000, p.293). Carmona et al (2013) had noteworthy findings on the benefits of using Model Eliciting Activities to teach mathematics. They found that “students in the third grade were not only developmentally ready to construct mathematical ideas that could be generalized to situations beyond the current one, but their models embedded some of the very same powerful mathematical ideas contained in the models developed by the post- baccalaureate problems-solvers”(Carmona et al, 2013, p. 252). Through Model Eliciting Activities I expect to see the students generating mathematical ideas that they can apply to other problem situations. Carmona and Greenstein (2013) also found that “the processes by which each group arrived at its solution were qualitatively distinct in the initial ideas that were expressed, the means they used to test those ideas, and the criteria they used to revise them” (p. 253). By using a Model Eliciting Activity I hope to help the students explore different processes to arrive at an answer rather than giving them one linear way of solving a problem.

Finally, a study conducted by Steffen Iverson and Christine Larson (2008) found that there was no statistically significant correlation between a students’ pretest and their MEA score (2008). This suggests that MEAs are an activity in which all students can participate and thrive in even if they are lacking in prior knowledge. Iverson and Larson (2008) also found that neither the pre-test nor the post-test did a good job of revealing which students were able to do well when working with the MEA problem. I want to make sure that I am able to gain some insight as to how the MEA shaped the student’s understanding of the mathematics so I chose to avoid pre and post tests and use clinical interviews instead.

Clinical Interviews

In order to assess student's growth in understanding this study will utilize clinical interviews to assess their knowledge before and after the activity. In a clinical interview the interviewer poses a series of questions that are intended to look inward to see how the child is thinking. It "can provide both researcher and practitioner with deep insights into a child's thinking" (Ginsburg 1997, p.2). I chose to conduct clinical interviews because it can be a "powerful vehicle for entering a child's mind" (Ginsburg, 1997, p. iv,). In employing the clinical interview, the examiner- practitioner or researcher- literally treats each child differently. Indeed, the clinical interview is deliberately non-standardized, thus violating the central tenet of testing" (Ginsburg, 1997, p.2). This individualized testing strategy will allow me to see what the students know and, if they don't know something, it will help me to see what they do think. A clinical interview gives much more depth and insight to a student's understanding than a standardized test.

CHAPTER 3: METHODS

Project Design

The Gazebo Project was created out of an effort to close gaps in students' fractional knowledge. The Gazebo project was not modeled after any one template but it was inspired by research on open-ended generative learning models and model eliciting activities. Model Eliciting Activities "not only help to document development; they also promote development" (Lesh,2000, p 594) and this became a central theme in The Gazebo Project. Like a Model Eliciting Activity, the Gazebo project was designed to promote students development in their understanding of fractions, not simply to assess their understanding of fractions.

The Gazebo Project
Woody's wood shop makes beautiful gazebos. They can make gazebos in the shape of any polygon. Regardless of the shape of the gazebo they only make gazebos with a perimeter of 60 feet. Each side of the shape makes a bench but one side has to be left open as the entrance for people to come in and out of the gazebo. Del Valle Middle School wants to buy a gazebo. They want the gazebo to have as much seating as possible so they want the opening to be as small as possible. Design a gazebo for Del Valle Middle School that will maximize their sitting space and minimize the length of the entrance. You may use the tickets to represent the perimeter of the gazebo. Each ticket represents one foot of wood.

Figure 1: The Gazebo Project

Reflection Questions
1) What shape did you create for the Gazebo? Why? 2) In your shape, what fraction of the perimeter was used for the entrance? 3) What part of the fraction is represented by the entrance? 4) What part of the fraction is represented by the number of sides of the polygon? 5) What did you have to do to minimize the length of the entrance?

Figure 2: The Gazebo Project Reflection Page

Student Selection

Eight students were selected from a sixth grade teacher's class in a Middle School in Texas. The school is a title one school consisting of around 960 students and 320 6th grade students. The teacher was asked to select four students who are high performing and four students who are low performing. The teacher was asked to take into account the students' fractional reasoning when making the selection. Four males were selected and four females were selected. IRB protocol was followed when selecting the students and permission was given from the district.

Grouping of Students

Four of the students were placed in a group setting. The group consisted of two high performing students and two low performing students. The group setting was designed to be collaborative. All students were encouraged to participate equally and to contribute in the creative process. The four students collaboratively answered the reflection questions at the end of the project.

Two of the students were placed in a peer tutoring setting. The peer tutoring section consisted of one high performing student who was assigned to be the peer tutor, and one low performing student who was assigned to be the tutee. The students were informed before beginning the project that they were in a peer tutoring section and they were informed who was labeled the tutor and who was labeled the tutee. The peer tutor was asked to tutor the other student and instructed to help the tutee understand. The tutee was told that the tutor was there to work on a group project with her. She was asked to participate and to let him know if there was anything she did not understand. The students answered the reflection questions in the group of two, although the peer tutor played more of a teacher role.

Two of the students did the project independently. One of the students was a high performer and the other student was a low performer. The students worked on the project and the reflection questions individually.

Task

The students were allotted one fifty minute class period to work on the project. The project was personally conducted during the allotted conference. Students were pulled out of their class to work on the project. The students were pulled out of the class one at a time for the clinical interviews while the other subjects remained in their class until they were ready to be interviewed and ready to begin the project.

Process

All of the students were given a pre-interview. The pre-interview began with one central question; if you were to compare the fractions one out of three and one out of fifty which one would be bigger and why? They were tested through the form of a clinical interview that allowed for a greater insight to the students' understanding of fractions. After the students selected the fraction they believed was bigger they were asked to explain their logic. The pre-project clinical interview was given individually to ensure that students could not hear each other's responses.

Immediately following the clinical interview pre-assessment the students were placed into one of the three sections; group, peer tutoring, and individual where they worked on the project and answered the reflection questions. Before they were released to work on their projects they were given sixty tickets that were taped at the end to make a large circle. The sixty tickets were given as a manipulative representing the perimeter of sixty feet. The students were then informed verbally that regardless of the shape they selected they would have to keep the perimeter uniform at sixty feet. They were also informed verbally that they would have to keep all sides of their design the same length for whatever shape they selected. Sixth grade students have not learned about regular polygons in the curriculum so the concept was briefly explained. At this point they were asked to begin working on the project.

Following the project the students were given a post project clinical interview. The post project clinical interview was given individually to ensure that students could not hear what their peers were saying. When conducting the post project interview the question was carefully phrased to avoid leading the students to doubt their original answer. They were asked the same central question; If you were to compare the fractions one out of three and one out of fifty which one would be bigger and why? The central question was again followed by questions about their logic and reasoning in their selection.

All of the interviews were recorded and transcribed and can be found in Appendix A.

Chapter 4: RESULTS AND DATA ANALYSIS

Group Section Observations and Analysis

Project Observations

The group work section had difficulty identifying the word gazebo. It was explained to them verbally. One of the students then thought about pulling up pictures on his smart phone. After seeing different examples the students all understood what the word meant. Some of the students struggled with the concept of regular polygons but through the process of designing irregular polygons and being corrected by their peers they were able to come up with a regular polygon by the time they turned in their project. Some students struggled to read the prompt and understand the goal of the project. Basic reading and problem solving skills were difficult for the low performing students. They overlooked the sentence that said that “they want the gazebo to have as much seating as possible so they want the opening to be as small as possible”. They were focused on creating a gazebo that would have shade until they received help reading the prompt from the teacher. One of the low performing students had a very difficult time concentrating during the project. He was looking around the room and copied down most of what the other students said without asking questions or participating. He needed a lot of supervision in order to participate and even then did not contribute at the level of the other students. He showed gaps in his understanding of fractions during the pre-project interview and showed those same gaps in the post-project interview as is evident in the transcribed clinical interview. This is an example of a student who may not benefit from a group environment where it is possible to complete the project with the same answers as your peers while failing to understand the material yourself. This subject displays the importance of participation in learning. It also demonstrates the importance of teacher supervision in generative group projects to help ensure that all students are encouraged to participate in order to increase the effectiveness of the project. This can be difficult in large classes where teachers have to split their time among five or six different student groups.

The other three members of the group contributed equally in the design process. The group section created a decagon which they called a “tenagon” for their gazebo. Their reflection questions were accurately filled in. They expressed that one-tenth of the perimeter was used for the entrance. They also stated that they would need to add more sides if they wanted to minimize the length of the entrance and that they would have to “make less sides” if they wanted to make a large entrance.

<u>Clinical Interview Observations</u>

The following are alias student names.

Group Section 1st High Performer:

Dana came into the project understanding that one out of three is a bigger fraction than one out of fifty. Dana demonstrated knowledge of fractions in the pre-project interview. She was able to determine that the fraction one out of three was bigger than the fraction one out of fifty. She demonstrated that she was able to identify the bigger fraction by picturing it when she said “because if you draw it, when you compare it, you will see one out of fifty is bigger”. She participated in the project and discussed the reflection questions with her peers. In the post project she was able to draw connections between the concept of fractions and the gazebo project. She was able to connect that that the relationship between the entrance and the total number of sides could be represented as a fraction.

Group Section 2nd High Performer:

David came into the project with a great deal of understanding about fractions. In the pre-project interview he was able to quickly convert one out of fifty to two percent and he was able to make the connection that since two percent is very small one out of fifty would likely be the smaller fraction. Throughout the project David helped guide his peers. He listened to their thoughts and then helped them to think about new ideas. David played a big role in evaluating his peers’ theories and challenging those theories to help create a more efficient gazebo. In the post project interview David was able to explain his reasoning for choosing one out of three as the bigger fraction.

Group Section 1st Low Performer:

During the pre-project interview Ellen demonstrated gaps in her understanding of fractions. She stated that one out of fifty was bigger than one out of three because “fifty is more than three”. She was not able to speak eloquently about the parts of the fractions or make any other observations about the two fractions. During the project she participated and gave ideas. In the post project interview Ellen had changed her perceptions about fractions and stated that one out of three was bigger than one out of fifty. She spoke about the project in her explanation. It seemed that she was trying to say that in the project when they wanted to make the entrance smaller they needed to have more sides but she had difficulty articulating her thoughts.

Group Section 2nd Low Performer:

Tim displayed gaps in his knowledge of fractions in the pre-project interview as he believed that the fraction one out of fifty was bigger than one out of three because fifty is bigger than three. He did not participate during the project but rather seemed to simply write down what his peers were saying without commenting. He showed the same gaps in knowledge in the post-project interview.

<u>Group Section Analysis</u>

The group section created the strongest gazebo with the most seating and smallest entrance. Through the process of designing the gazebo the students suggested ideas for the shape of the gazebo and through discourse were challenged to increase the number of sides in the shape to a ten-sided figure. As Walter Stroup et al. (2007) suggested the students were able to “express, test and revise their own ways of thinking” (p. 367) and as a result increased the number of sides until they reached a decagon. They also used language as a tool to enable them to create a plan (Vygotsky, 1978, p.24). When the students had difficulty understanding what a gazebo was they helped each other understand through language and by finding pictorial examples to demonstrate what they were saying. The students were constructing theories about the gazebo (Lave et al., 1988, p71) and deconstructing each other’s theories. For example, some of the members of the group suggested making a six sided figure and then another member of the group suggested “well why not make a tenagon”? As a group they were able to create a superior design and answer the reflection questions, which were targeted at understanding the basic concepts in fractions.

Peer tutoring Section Observation and Analysis

Project Observations

The peer tutoring section also needed help identifying the word gazebo. They were given a verbal description and were then able to think of examples. The peer tutoring section came to their final design after a process of creating a design and then critiquing designs. At one point the peer tutor suggested making “something with 6 sides”. The tutee then interjected “or we could do something with more sides. It would make more seats and a smaller entrance”. The tutee then suggested an octagon and the group of two agreed on the octagon. While completing the reflection questions the tutee had a difficult time understanding number three; “In your shape what fraction of the perimeter was used for the entrance”? The peer tutor explained that the entrance was only one of the total sides making the fraction one-eighth. The peer tutor explained that you would have to add more sides in order to minimize the length of the entrance (reflection question 5) and that you would have to have less sides in order make a larger entrance (reflection question 6). When the tutee got to number eight on the reflection questions; “How does this relate to the topics you have studied this year in math class?” she was able to carry the concepts she learned from the project over to the reflection question. She stated that the project was related to fractions. When she was asked how it was related to fractions, she explained that “there are eight sides but we only have one side for the entrance so it shows a fraction, one out of eight”.

Clinical Interview Observations

Peer Tutor:

In the pre-project interview Brian showed understanding of fractions as he was able to speak eloquently about fractions. He was able to explain that one out of three was bigger than one out of fifty because the “slice is huge” compared to the “little pieces" in one out of fifty. In the post project interview Brian showed that he still understood why one out of three is the bigger fraction though he had a difficult time verbalizing his thought on the project.

Peer Tutee:

The pre-project interview exposed Clair's gaps in her knowledge of fractions. She believed that one out of three was smaller than one out of fifty because "the bottom is bigger" in one out of fifty. During the project Clair participated heavily and was already showing growth in her understanding before the end of the project. In the post-project interview Clair explained that one out of three was bigger than one out of fifty and used the project to explain her reasoning. Without any prompting Clair explained that one out of three was bigger because the smaller the entrance the more seating there is inside and the bigger the entrance the less seating there is inside. Clair was able to connect that as the number of sides increased, the size of the entrance would decrease and the seating would increase. She finally ended her interview with the statement "if you have less sides you have a bigger entrance" summing up her understanding of the project and its connection to the essential question. The gazebo project helped fill gaps in Clair's understanding of fractions and the peer tutoring setting allowed for Clair to verbalize thoughts and ask questions resulting in an increased understanding of fractions.

Peer Tutoring Section Analysis

The peer tutoring section displayed similar characteristics to the group section. They created the second best gazebo following the group section. After a process of "expressing, testing, and revising their own thinking" (Walter Stroup, 2007, p367) the peer tutoring section designed a gazebo in the shape of an octagon. It was very interesting to see that the tutee contributed as much as the tutor. The tutee did not show that she was intimidated to contribute or inhibited by her role as a tutee; rather, she spoke up and even suggested that she may have a better idea than the tutor. When the tutor presented the idea of creating "something with six sides" the tutee felt free to test the expressed theory and suggested a revised plan. She suggested "making something with more sides" and also explained her reasoning by stating that "it would make more seats and a smaller entrance". Once again we see the students verbalizing their thoughts, receiving critique on those thoughts, and then creating an improved plan. The tutee later needed the tutor to help her understand the third question, what fraction of the perimeter was used for the entrance, and the tutor was able to help her make the connection. Later when the pair approached number eight on the reflection questions, "how does this relate to the topics you have studied this year in math class" the tutee was able to make connections on her own. She found a relationship between the total sides to the one side used for the entrance. The tutor and tutee are working together and contributing equally even in the confines of a tutor tutee relationship showing perhaps more success than the group of four.

Individual Section Observation and Analysis

Project Observation

Both members of the individual section had a difficult time understanding the project. They needed the project explained to them several times before they could begin. Both the high performing student and the low performing student did not understand the concept of regular polygons. The low performing student turned in a final draft of a gazebo that was in the shape of a rectangle. He was given further explanation on the concept of regular polygons and asked to redo his project.

Clinical Interview Observation

Individual Low Performer:

During the pre-project interview Max displayed gaps in his understanding of fractions. He explained that one out of three was smaller than one out of fifty because one out of fifty "sounds bigger". During the project Max initially created a rectangular gazebo and turned it in as his final project. The misunderstanding was caught immediately and he was corrected about the concept of regular polygons and given a second chance to work on his project. The second time he did create a square. In the post-project interview Max showed no growth in his understanding of fractions. He still believed that one out of three was smaller than one out of fifty and he had a very difficult time verbalizing how the project related to fractions. Max did not show any benefit in participating in The Gazebo Project.

Individual High Performer:

Melissa showed an understanding of fractions in the pre-project interview. She explained that one-third was bigger than one out of fifty because one out of fifty is “separated into smaller pieces”. In the post-project interview Melissa was still able to answer that one out of three was the bigger fraction but when asked if the project had anything to do with fractions she answered, “kind of” and had difficulty drawing any connections. Also, though Melissa picked a hexagon, when asked why, she stated that having more sides would give you more options in picking a side for your entrance, “and not all of them are the same size” so “you can pick which one can be the least and which one would work for the entrance”. This statement exposed that Melissa did not benefit from the project. Without the realization that the gazebo had to be a regular polygon with sides of equal lengths there would be no connection to fractions which must have equal parts. Again, both members of the individual group made this mistake as opposed to the group and peer-tutoring sections which were able to verbalize and critique each other’s thoughts.

Individual Section Analysis

The individual section struggled in all aspects of the project. Both the high and low performing students went through the whole project with the misconception that the sides of the gazebo could be of different lengths, causing them to lose sight of the point of the project. This speaks volumes about the role of student discourse and group work in order to make open ended generative projects effective. Though members of the group section and the peer tutoring section had similar misconceptions about the project, both sections were able to clear up those misconceptions through discourse. As one student suggested a gazebo containing sides of different lengths another was present to test and revise their theory. Both students did not show any growth in their understanding of fractions from the pre-project interview to the post-project interview.

Conversely, two out of the three low performing students who were placed in group and peer tutoring settings started out the pre-project interview with gaps in their understanding of fractions and were able to close those gaps and verbalize what they had learned by the end of the project.

Chapter 5: CONCLUSIONS

The Gazebo Project exposes the effectiveness of open-ended generative model eliciting projects in helping students learn mathematical concepts but only when students work on such projects in a group setting as far as is evident from this study. This study also shows the benefits of using clinical interviews to regularly assess student understanding rather than relying on their answers for an assignment. Additionally, the study displays the effectiveness of both the group work section and the peer tutoring section, as both showed benefits to all parties involved.

The Gazebo Project built upon findings of the benefits of using open-ended model eliciting projects to engage students in learning. The Gazebo Project was designed to help students develop their own reasoning about comparing and ordering fractions. Though the project did not directly address this issue of comparing fractions, many of the students were able to make observations about the relationship between the number of sides in a fraction and the value of each side. Five out of six students that were placed in a group or peer tutoring setting were able to draw the conclusion that given a fixed perimeter, the value of each side in a shape increases as the number of sides decreases and the value of each side decreases as the number of sides increases. This is noteworthy as The Gazebo Project never explicitly taught that concept. Rather the students arrived at that realization through a process of creating an optimal design. Five out of six students were then able to take those findings and apply them to an explicit mathematics problem on comparing and ordering fractions, “If you were to compare one out for three to one out of fifty, which one would be bigger”.

For future experimentation I would like to adapt the post-project interview question. I would like the question to address the same concept but with different numbers. This will help to ensure that the students are not simply reversing the answer they chose in the pre-project interview. The clinical interviews did allow for insight into the logic that the students used when selecting their answer, however changing the post-project interview question would be a good precaution to ensure that the students did close gaps in their understanding.

Using clinical interviews proved to be very effective in accessing insight to the ways that students were thinking. If assumptions were drawn from the ways that students answered the reflection questions, one would be lead to believe that many of the students closed gaps in their fractional reasoning that in fact had not. The clinical interviews allowed for insight to the child's reasoning and explanation, exposing gaps in knowledge or misunderstandings about the project. Clinical interviews not only help educators better understand their students understanding but also can be used as a tool to improve curriculum. For example, the post project clinical interview exposed that students in the individual section did not understand the concept of regular polygons or a constant perimeter, resulting in minimal benefit from the project. Through the clinical interview that issue was realized, allowing the educator to address the potential problem for the next administration of the project or lesson. Teachers can sometimes be deterred by the clinical interview process because of the time they take to administer. Due to the time constraints in the students' schedules, the clinical interviews that were used in this project were quite short compared to typical clinical interviews and required little time to administer yet they were very effective in gathering knowledge about what the students were thinking and misperceiving. This study suggests that even a short clinical interview can be effective in gathering useful data.

The findings show that only the students that were placed in the group section or the peer tutoring section were able to connect the project to the post project mathematical interview. The students who were placed in the group and peer tutoring section used "language as a tool" to help them with problem solving (Vygotsky, 1978, p. 19). The Gazebo Project coincided with the observations of Walter Stroup (2007) that "students generate knowledge by repeatedly expressing, testing and revising their own ideas" (p. 367). During The Gazebo Project, the students who were placed in groups discussed their designs, drew their designs, critiqued each other's designs and then revised their designs. When the students were able to work in groups, they were "engaging in choice, judgment … problem formation and in making wrong choices" (Lave et al., 1988, p. 77). The student's benefited from making even incorrect choices, as long as they were making them with peers that could critique and revise their plan. These findings confirmed the hypothesis that students spawn understanding through a process of verbalizing their thoughts and receiving critique on those thoughts from peers. One reason that this may be true is because “in group problem solving sessions, it is natural for students to externalize ways of thinking that might remain internal otherwise” (Kieren, 2000, p.229).

The students in the individual section did not have the opportunity to externalize their ways of thinking and therefore completed the project with some fundamental misunderstandings. Both students did not grasp the concept that the figures needed to be regular polygons and that the perimeter was a constant. The students in the group sections were able to clear up their misconceptions by being corrected by their peers when they verbalized a flaw in their logic. The students in the individual section did not have that opportunity to externalize their thoughts. For a future study it would be beneficial to make sure that the definition of a regular polygon is written in the prompt so that the students can see the prompt in addition to having the administrator explain the concept.

The Gazebo Project exposed the benefits not only of group work but of the peer tutoring model. The students placed in the peer tutoring model experienced similar benefits of student discourse as the group project section. Both the peer tutoring section and the group section were able to clarify their peers’ misunderstandings. Both peers showed a process of expressing, testing and revising their ideas and both sections used language as a tool. The peer tutoring section did not show any signs of the students being inhibited by their roles. Both the peer tutor and tutee made contributions and we found that the peer tutee even felt comfortable to revise the tutor’s idea with an improved suggestion.

Open-ended generative projects, including model eliciting projects are highly effective in enabling students to come to their own revelations about mathematical concepts. Their non-prescriptive nature allows for students to make their own discoveries about mathematics rather than being guided to a predetermined solution. Open ended, generative activities “reveal how they are interpreting the mathematical situations that they encounter” (Lesh, 2000, p.293). The Gazebo Project exposed that a key element to a successful open-ended generative project is student discourse. This study makes it evident that the opportunity for learning is greatly increased when open ended generative activities are paired with the “expressing, testing, and revising” (Stroup, Ares, Hurford, & Lesh, 2007, 367) that comes with a student’s ability to work in groups rather than in isolation. Through discourse students are able to create a revised and improved plan (Vygotsky, 1978, p.19, 24) and in doing so they are able to reach more understanding as a community than other students in the study reached on their own.

Appendix A

Transcription of Clinical Interviews

Group Project 1st High Performer Pre-project Data:

Natasha- I have one main question for you. If you were to compare the fractions one-third and one- fiftieth, which one would be bigger (I then showed a piece of paper with the fractions written out numerically).

Dana- One third

Natasha- One third? Why do you think one third is bigger?

Dana- Because if you draw it one out of three is the biggest and one out of fifty is the smallest.

Natasha- Why is that?

Dana- Because if you draw it, when you compare it you will see one out of fifty is the smallest.

Natasha- Ok, thank you.

Group Project 1st High Performer Post-project Data:

Natasha- OK, now that you have had time to work on this project I am going to ask you the same question about fractions. Which one is bigger? One out of 3 or one out of fifty?

Dana- One out of three.

Natasha- Why do you say that?

Dana- Because if you draw the squares of one-third and one out of fifty, one-third is bigger.

Natasha- And how does this relate to the project we did?

Dana- Because we had to find the fraction of the entrance to the gazebo.

Natasha- You had to find the fraction of the entrance to what part of the gazebo? All fractions have two parts.

Dana- To the total gazebo.

Natasha- Thank you

Group 2nd High performer Pre-project Data:

Natasha- I have one main question for you. If you were to compare the fractions one-third and one-fiftieth, which one would be bigger (I then showed a piece of paper with the fractions written out numerically).

David- One out of three.

Natasha- Ok, and can you tell me why?

David- There are only three pieces and one of them is shaded so that's one third of it and then with one out of fifty one is like two percent.

Natasha- Ok, any other thoughts you would like to share?

David- No

Group 2nd High Performer Post- project:

Natasha- OK, now that you have had time to work on this project I am going to ask you the same question about fractions. Which one is bigger? One out of 3 or one out of fifty?

David- One out of three.

Natasha- Ok, can you tell me why.

David- Because one third has just three pieces of it and its split into three pieces and one out of fifty has fifty pieces and so all the pieces are tiny.

Group 1st Low Performer Pre-project Data:

Natasha- I have one main question for you. If you were to compare the fractions one-third and one-fiftieth, which one would be bigger (I then showed a piece of paper with the fractions written out).

Ellen- One out of fifty.

Natasha- You think one out of fifty is bigger?

Ellen- Yes because fifty is more than three and one, one is the same so you have to say that fifty is bigger than three.

Natasha- Ok, have you ever compared and ordered fractions before?

Ellen- What do you mean?

Natasha- Have you ever ordered fractions from biggest to smallest?

Ellen- Yes

Natasha- Ok, do you have any other observations about the fractions? Any observations about the parts of the fractions?

Ellen- No

Natasha- Ok, thank you.

Group 1st Low Performer Post-project Data:

Natasha- OK, now that you have had time to work on this project I am going to ask you the same question about fractions. Which one is bigger? One out of 3 or one out of fifty?

Ellen- One out of three.

Natasha- You think one out of three is bigger? Are you sure about that?

Ellen- Yes

Natasha- Why?

Ellen- Because of the tickets. We tried to make the door smaller and three is smaller than fifty.

Natasha- Can you elaborate on that?

Ellen- On the ticket thing we tried to make the door smaller. The total was sixty so I think one third is bigger than one out of fifty.

Group 2nd low performer Pre-project Data:

Natasha- I have one main question for you. If you were to compare the fractions one-third and one-fiftieth, which one would be bigger (I then showed a piece of paper with the fractions written out numerically).

Tim- One out of fifty.

Natasha- Can you tell me why.

Tim- Because fifty is greater than three.

Natasha- Ok, how do fractions work?

Tim- When you have five squares and two pieces it's two fifths.

Natasha- Ok, and what are the parts of a fraction?

Tim- I don't know

Natasha- Ok, that's alright. Thank you.

Group- 2nd Low Performer Post-project Data:

Natasha- OK, now that you have had time to work on this project I am going to ask you the same question about fractions. Which one is bigger? One out of three or one out of fifty?

Tim- One out of fifty.

Natasha- Why is that?

Tim- Because fifty is bigger.

Natasha- In your project you were asked to compare two different gazebos and you tried to make the door as small as possible. What did you do to make the door as small as possible?

Tim- We chose a tenagon instead of a triangle.

Natasha- Why did you choose a tenagon?

Tim- The shape is bigger and you have more sides and more inches to come in it.

Natasha- Ok, thank you.

Peer Tutor High Performer Pre-project Data:

Natasha- I have one main question for you. If you were to compare the fractions one third and one out of fifty, which one would be bigger (I then showed a piece of paper with the fractions written out).

Brian- One third

Natasha- One third? Why is that?

Brian- Because it’s cut up into bigger pieces.

Natasha- Can you elaborate on that a little bit for me?

Brian- It’s kind of hard to explain it.

Natasha- If you were to explain it to your little sister how would you explain it?

Brian- Like I would say one out of fifty are like a bunch of little pieces that are cut up into fifty and only one slice. That’s why it’s really small compared to one out of three where the slices are huge.

Peer Tutor High Performer Post-project Data:

Natasha- Alright Brian, now that you have had time to do some peer tutoring do you have any other thoughts on one third versus one out of fifty?

Brian- No

Natasha- You still think one third is bigger?

Brian- Yes

Natasha- Can you tell me one more time why one third is bigger and think a little bit about the project you just did?

Brian- One third is bigger because one out of fifty the pieces are smaller, just like the tickets thing and in one out of three the pieces are bigger.

Natasha- Thank you very much.

Peer Tutor Low Performer Pre-project Data:

Natasha- I have one main question for you. If you were to compare the fractions one third and one out of fifty, which one would be bigger (I then showed a piece of paper with the fractions written out).

Clair- One out of fifty

Natasha- Why do you think one out of fifty is bigger?

Clair- Because the bottom one is bigger.

Natasha- Ok, is there any other reason that you think one out of fifty is bigger?

Clair- No.

Natasha- Ok, thank you.

Peer Tutor Low Performer Post-project Data (Clair):

Natasha- Alright, so now that you have done the project think about the original question that we had. Which one is bigger, one out of three or one out of fifty?

Clair- One out of three because the smaller the entrance the more seating there is inside and the bigger the entrance the less seating there is inside.

Natasha- And how does the project we just did relate to fractions?

Clair- If there are eight sides and you only need one entrance then it would be one out of eight and there would be eight sides in all. If it has more sides you have more seating and a smaller the entrance, if you have less sides you have a bigger entrance.

Natasha- Thank you.

Individual Low Performer Pre-project Data:

Natasha- I have one main question for you. If you were to compare the fractions one-third and one- fiftieth, which one would be bigger (I then showed a piece of paper with the fractions written out numerically).

Max- One out of 50

Natasha- Why is that?

Max- Because it sounds bigger.

Natasha- you think it sounds bigger. What sounds bigger?

Max- One out of fifty.

Natasha- Ok, and to make sure you can visualize the fractions here is one out of 3 written out and here is one out of 50 written out (and pushes the sheet of paper toward him). Do you still believe 1/50 is bigger?

Max- Yes

Natasha- Ok, thank you.

Individual Low Performer Post-project Data:

Natasha- OK, now that you have had time to work on this project I am going to ask you the same question about fractions. Which one is bigger? One out of 3 or one out of fifty?

Max- One out of fifty.

Natasha- Ok, and why do you think one out of fifty is bigger?

Max- Because on the bottom it says fifty and fifty is bigger than three.

Natasha- Ok, how did the problem we just did relate to fractions?

Max- Um, a little bit hard.

Natasha- Did the project have anything to do with fractions?

Max- Yes, the size and the shape. You had to figure out how many seats and you only had one square so that's mostly a fraction.

Natasha- So why did you chose a square?

Max- Because on one side you could put twenty, so you could put sixty chairs.

Natasha- Ok, thank you.

Individual High Performer Pre-Project Data (Alyssa):

Natasha- I have one main question for you. If you were to compare the fractions one third and one out , which one would be bigger (I then showed a piece of paper with the fractions written out numerically).

Melissa- One third.

Natasha- Why is that?

Melissa- Because it is separated into smaller pieces so one out fifty is going to be a smaller portion because it is out of fifty.

Natasha- Because it's out of fifty? Can you elaborate on that?

Melissa- Um, one out of fifty is just a small portion of it, but one out of three is a bigger portion because it has a smaller number.

Individual High Performer Post-project Data (Alyssa):

Natasha- Ok Melissa, what shape did you pick?

Melissa- I picked a hexagon.

Natasha- And why did you pick a hexagon?

Melissa- Because it has more sides and it would be easier for you to narrow down what side is the entrance because it has more sides so you can pick which side would be the best.

You can narrow down your options. Like if you have six sides and not all of them are the same size you can pick which one can be the least and which one would work for the entrance.

Natasha- Ok, and going back to our earlier question, if you had to pick which one is bigger, one out of three or one out of fifty which one would you pick?

Melissa- One out of three.

Natasha- And why is that?

Melissa- Because it is cut into big pieces and if you shade in one it would be a bigger piece than one out of fifty. If you had to cut it into fifty pieces it would be least because there are so many pieces.

Natasha- So, do you think that this project has anything to do with this question about comparing one third to one out of fifty?

Melissa- Kind of. Because you are comparing fractions and in both of them you are trying to find a way that you can get a fraction out of it. You are comparing both of them. In the hexagon you are trying to find the one that has the least amount so that it has more space. You are trying to compare which one is bigger and which one is smaller.

Natasha- Ok, thank you.

Appendix B

The knowledge I have gained throughout my experience in the UTeach Engineering MASEE program has prepared me to be a strong teacher and resource to fellow teachers. I have been exposed to pedagogy that has dramatically shaped the way that I teach. For example, the clinical interview process has enlightened me to the ways that my students are reasoning and has given me insight to their thought processes, better equipping me to bridge gaps in their understanding. The clinical interview enables me to receive feedback from my students at the end of each lesson, helping me to constantly improve my lessons and address misconceptions.

As a middle school math teacher, one of the greatest gaps I see in student understanding is the gap in fractional reasoning. Many students cannot explain why one fraction is bigger than another and struggle to visualize what fractions look like. This project gave me the opportunity to design a project that showed to be quite effective in bridging those gaps in fractional reasoning and it will be a project that I am able to share with my future students and colleagues.

I have been very passionate about the benefits of student discourse and peer tutoring throughout my career and it was very exciting for me to engage in a study that further explored the process. Again, this study has encouraged me to continue to encouraging group work in my class and on my campus and has reminded me to monitor closely and follow up with a clinical interview for best results in student achievement.

My experience having taught sixth, seventh and eighth grade mathematics curriculum has given me insight and appreciation for the ways that our curriculum can be applied to the public school classroom while also preparing me for practical challenges that will occur in the classroom. In addition, my background in social work as a youth care counselor at a residential treatment center for abused youth and a long term teacher in a title one school district opened my eyes to the ways that the curriculum taught through the MASEE program can challenge and inspire youth with challenging circumstances, opening many doors for their future.

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